A thermo-economic model for aiding solar collector choice and optimal sizing for a solar water heating system

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Presentation Layout

1. Introduction
2. Solar Water Heating System Thermal Model
3. Materials and Methods
4. Results and Discussion
5. Conclusion

Introduction (Motivation)

- **Worldwide, the heating of water to low and medium temperatures using solar thermal energy has gained popularity for many applications**
- **This is because of the favourable characteristics of solar water heating, which result in economical and environmentally-friendly displacement of conventional energy sources**
- **Solar thermal water heaters are a prudent option where:**

❑ **the cost of conventional energy for heating water is higher than $0.034/kWh;**

❑ **daily average solar irradiation is higher than 4.5 kWh/m^2 and**

❑ **where energy security is important (e.g. where there is interruptible supply of conventional energy) [1]**

ALL THESE PRE-CONDITIONS ARE APPLICABLE IN MOST OF SOUTHERN AFRICA

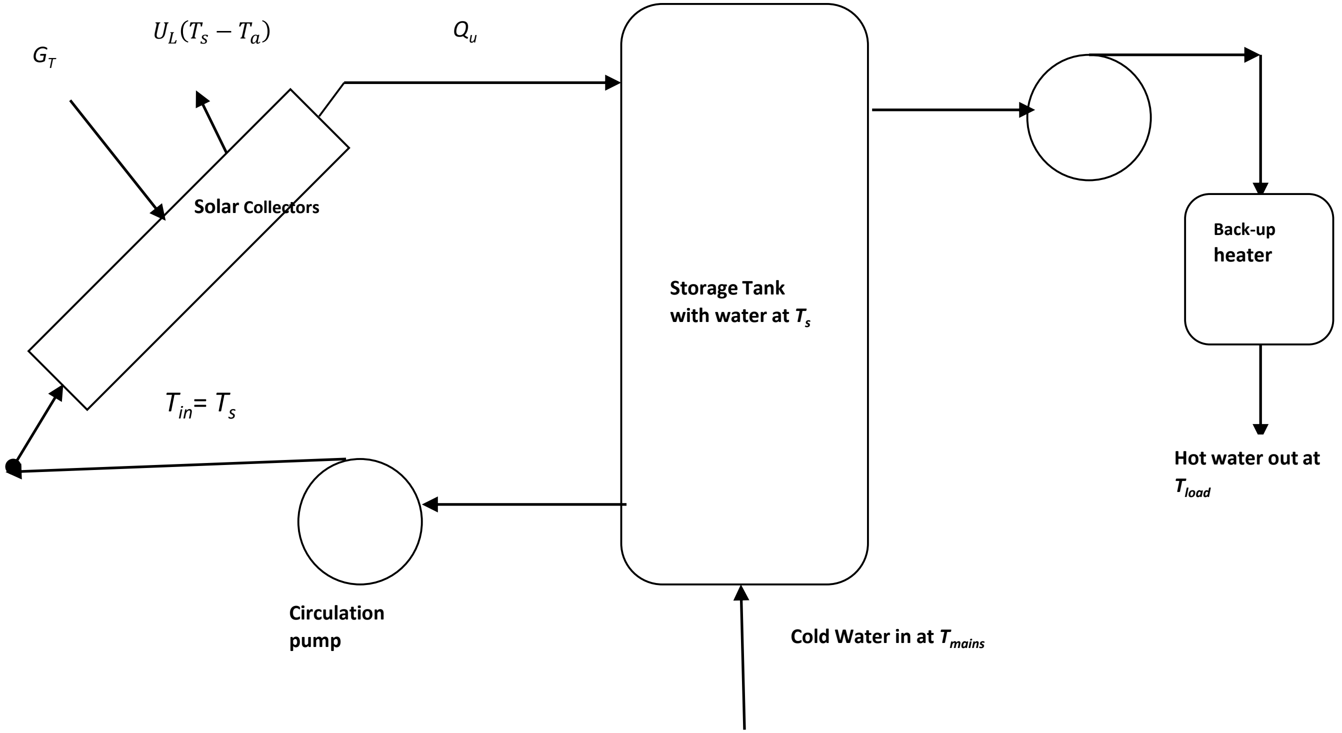
Introduction (Problem)

- In order to maximize the economic benefits derived from the use of solar water heaters, prudent choice of type and size of collector to employ is necessary.
- The various metrics ordinarily used to appraise the appropriateness of collectors, such as cost of collector, efficiency, warranty, are not adequate on themselves to give an objective basis for choosing between different collectors.
- A more objective decision metric, which is inclusive of an important few of collector attributes, is required for prudent selection of type of collector to use in a given water heating application.

Introduction (Study objective)

- To develop a thermo-economic model that can be used to guide the selection of the brand of solar collector(s) to be used in a solar water heating application, for a given required hot water temperature and under given climatic conditions.
- To define and apply an objective metric for appraisal of solar collectors:- *the energy-per-dollar metric.*
- To rank different models of flat plate and evacuated tube collectors by the energy-per-dollar metric, when needed for solar water heating applications under given climatic conditions
- To use the thermo-economic model for determining the optimal size of the choice collector area to deploy in the solar water heating system, together with corresponding required volume of hot water storage tank and solar fraction at optimal collector size.

Solar Water Heating System Thermal Model



EQUATIONS OF THERMAL MODEL

1. Temperature in storage tank

$$T_s^{+} = T_s + \frac{\Delta t}{(MC_P)_s}\left(Q_u - \dot{L}_s - U_s A_s[T_s - T_a]\right)$$

T_s^{+} = temperature at subsequent period	[^{o}C]
T_s = temperature at current period	[^{o}C]
M = mass of storage tank contents	[kg]
Δt = time period increment (e.g. 1 hour)	[s]
C_P = specific heat capacity	[J/kg/ ^{o}C]
Q_u = collector heat output	[J/s]
$\dot{L}_s$ = rate of heat removal	[J/s]
$U_s A_s$ = storage tank area-heat-loss-coefficient product	[W/ ^{o}C]
T_a = ambient temperature at current hour	[^{o}C]

EQUATIONS OF THERMAL MODEL

2. Collector heat gain

$$\boldsymbol{Q_u = A_c\{G_T K_{\tau\alpha} F_R(\tau\alpha)_n - F_R U_L(T_s - T_a)\}}$$

$\boldsymbol{Q_u}$ = collector rate of heat output (+ve only)	[J/s]
$\boldsymbol{A_c}$= collector area	[m^2]
$\boldsymbol{G_T}$= in-plane solar irradiance	[W/ m^2]
$\boldsymbol{K_{\tau\alpha}}$= incidence angle modifier	[-]
$\boldsymbol{F_R(\tau\alpha)_n}$= optical efficiency at normal incidence	[-]
$\boldsymbol{F_R U_L}$= heat loss parameter	[W/ m^2/^{o}C]

EQUATIONS OF THERMAL MODEL

3. Heat withdrawal and solar fraction

$$\dot{L}_s = \dot{m}_s C_P (T_s - T_{mains})$$

and

$$\dot{m}_s = MIN\left\{\frac{T_{load} - T_{mains}}{T_s - T_{mains}}; 1\right\} \times \dot{m}_{load}$$

$\dot{m}_s$= required mass of water withdrawal [kg]

T_{mains}= cold water from municipality mains [°C]

T_{load}= hot water temperature required by load [°C]

$\dot{m}_{load}$= mass of water required at temperature T_{load} [kg]

If $T_s > T_{load}$ then $m_s < m_{sload}$, else $m_s = m_{load}$

$$Solar\ Fraction, S_F = \frac{\dot{m}_s (T_s - T_{mains})}{\dot{m}_{load} (T_{load} - T_{mains})}$$

Energy-per-dollar

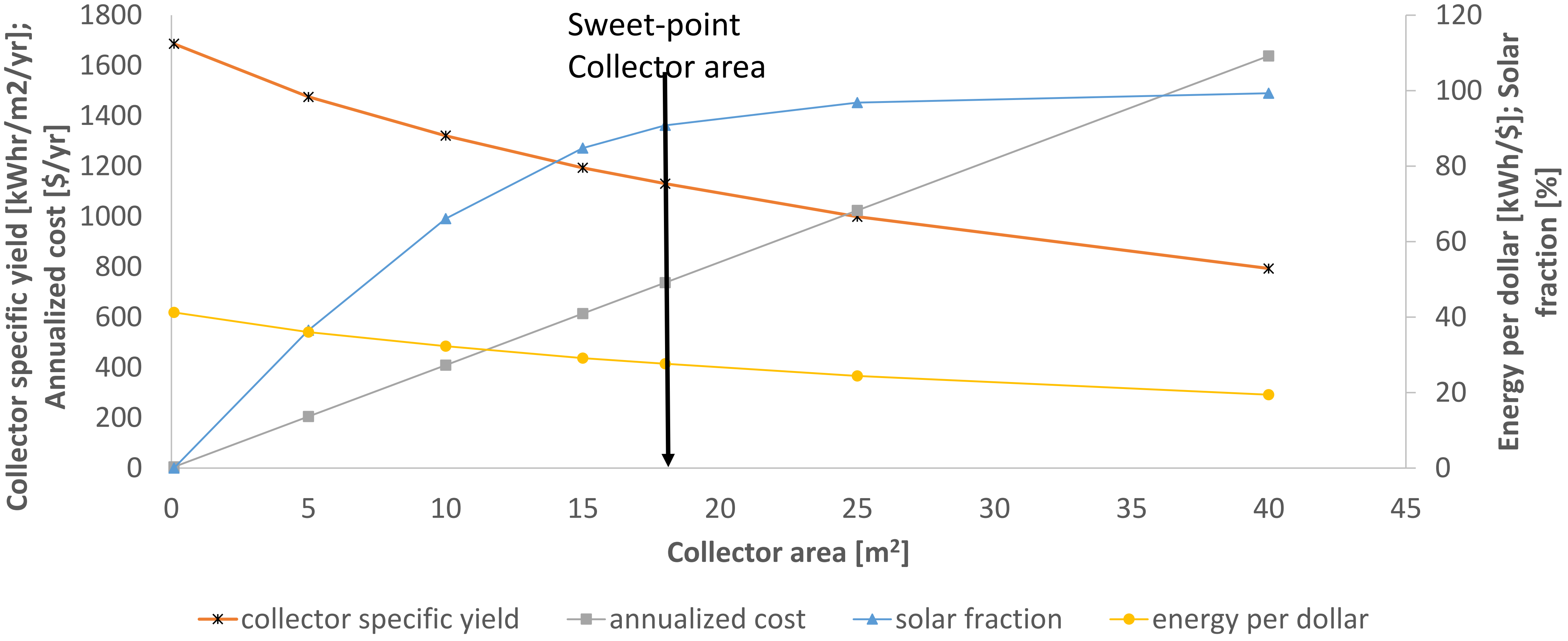
- **The *energy-per-dollar* of the solar collector is computed by dividing the *annual energy delivered by collector*, which is obtained from the thermal model, by the *annualized collector costs*, calculated for the *warranty* period.**
- **Annualized costs:** $C_{annual} = A_c C_c \frac{d}{1-(1+d)^{-w}} + OM$
- **Annual Energy:** $Q_{annual} = \sum_{year} L_s$
- $Energy - per - dollar = {Q_{annual}}/{C_{annual}}$

❖ A_c = area of collector [m2]; C_c= **cost of collector per m²**

❖ d **= discount rate [years];** w **= warranty life [years]**

❖ OM**= annual operation and maintenance cost [$]**

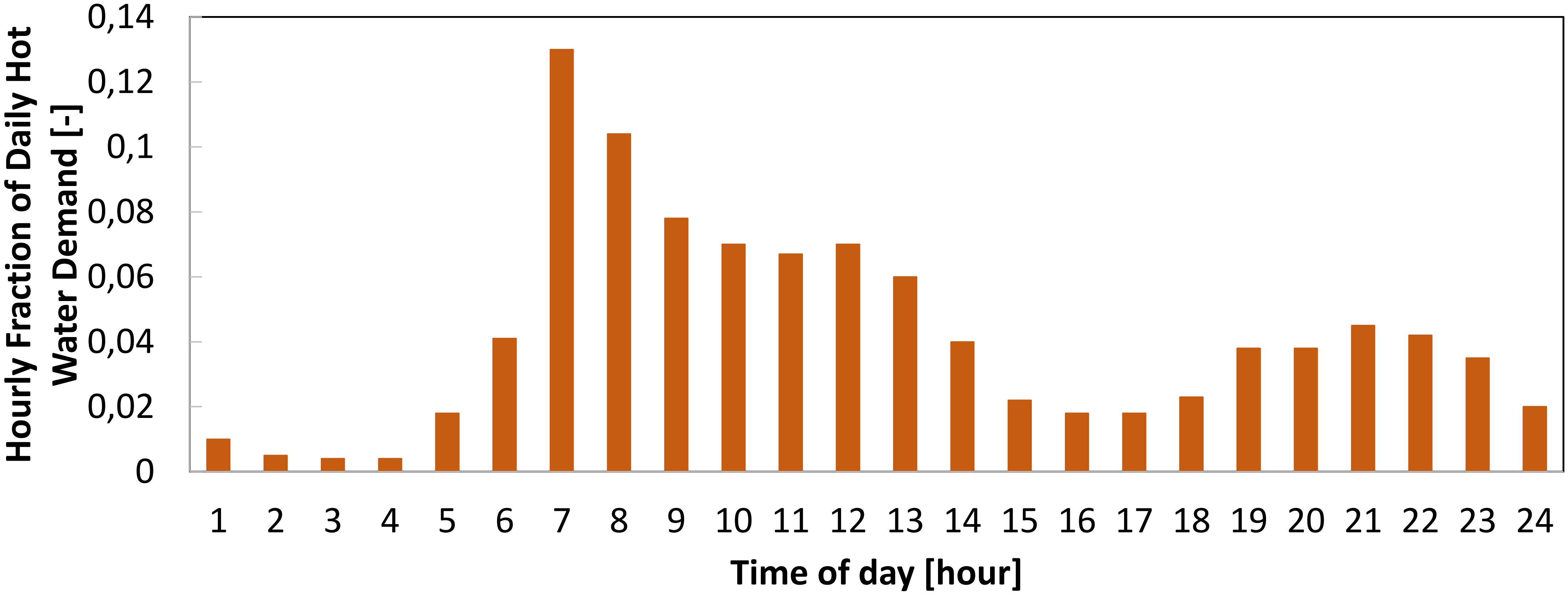
Diminishing marginal returns on energy-per-dollar



Materials & Methods: Climatic Data

Monthly average meteorological data for Kwekwe, latitude 19º S and longitude 30º E												
Month	January	February	March	April	May	June	July	August	September	October	November	December
Avg. Temperature (°C)	21.9	21.7	20.9	19.8	16.8	14.2	14.2	16.5	20	22.8	22.3	22
Min. Temperature (°C)	16.3	16	14.5	12.6	8.6	6	5.6	7.6	11.2	14.8	16	16.4
Max. Temperature (°C)	26.4	26	25.3	24.6	22.5	20.3	20.3	22.9	26.7	29.1	27.4	26.4
Global Horizontal Irradiation [kWh/m²/day]	6.65	6.48	6.35	5.83	5.27	4.90	5.07	6.02	6.83	7.18	6.84	6.35
Diffuse Horizontal Irradiation [kWh/m²/day]	2.4	2.3	2.0	1.5	1.2	1.1	1.0	1.2	1.5	2.0	2.4	2.6

Materials & Methods: Hot Water Demand Profile



Materials & Methods: Procedure for Calculating System Performance

Hour	G_T [W/m²]	Ta [°C]	Ts [°C]	$K_{\tau\alpha}$ [-]	Q_u [Wh]	$UA_s(T_s-T_a)$ [Wh]	m_{load} [kg]	Load [Wh]	m_s [kg]	ΔT_s [°C]	L_s [Wh]	Solar Fraction [-]
0 –1	0	20.0	52.4	0.000	0	473	10	401	9	-0.8	401	1.000
1-2	0	17.5	51.6	0.000	0	499	5	200	5	-0.6	200	1.000
2-3	0	14.6	51.0	0.000	0	531	4	160	4	-0.6	160	1.000
3-4	0	11.9	50.4	0.000	0	562	4	160	4	-0.6	160	1.000
4-5	0	9.8	49.8	0.000	0	583	18	721	18	-1.1	716	0.993
5-6	0	8.7	48.7	0.000	0	583	41	1642	41	-1.9	1578	0.961
6-7	66	8.8	46.8	0.522	0	555	130	5208	130	-4.5	4722	0.907
7-8	257	10.0	42.2	0.859	657	471	104	4166	104	-2.6	3226	0.774
8-9	473	12.2	39.6	0.921	3857	401	78	3125	78	1.1	2181	0.698
9-10	682	14.9	40.7	0.920	6522	377	70	2804	70	3.5	2047	0.730
10-11	847	17.8	44.3	0.950	8834	387	67	2684	67	5.4	2235	0.833
11-12	938	20.2	49.6	0.993	10318	429	70	2804	70	6.1	2772	0.989
12-13	938	21.9	55.7	0.993	10004	494	60	2404	51	5.8	2404	1.000
13-14	847	22.5	61.5	0.950	7951	570	40	1602	30	4.5	1602	1.000
14-15	682	21.9	66.0	0.920	5231	644	22	881	15	2.8	881	1.000
15-16	473	20.2	68.9	0.921	2364	710	18	721	12	0.5	721	1.000
16-17	257	17.8	69.3	0.859	0	753	18	721	12	-1.6	721	1.000
17-18	66	14.9	67.7	0.522	0	771	23	921	15	-1.9	921	1.000
18-19	0	12.2	65.8	0.000	0	784	38	1522	26	-2.6	1522	1.000
19-20	0	10.0	63.3	0.000	0	778	38	1522	27	-2.5	1522	1.000
20-21	0	8.8	60.8	0.000	0	759	45	1803	34	-2.7	1803	1.000
21-22	0	8.7	58.1	0.000	0	721	42	1682	34	-2.4	1682	1.000
22-23	0	9.8	55.7	0.000	0	669	35	1402	30	-2.0	1402	1.000
23-24	0	11.9	53.7	0.000	0	610	20	801	18	-1.3	801	1.000
SUM/AVERAGE	**6524**				**55738**	**14116**	**1000**	**40059**	**904**	**0**	**36382**	**0.91**

RESULTS

- Ranking of solar thermal collectors, rated by the Solar Rating & Certification Corporation(SRCC), according to the energy-per-dollar criterion
- Optimal sizing of the solar hot water system using the choice solar collector
  - Optimal collector area per m3 hot water demand
  - Optimal solar fraction
  - Optimal storage volume per m3 hot water demand
  - Maximum NPV per m3 of hot water demand
- Typical diurnal performance

Ranking of some SRCC-rated collectors

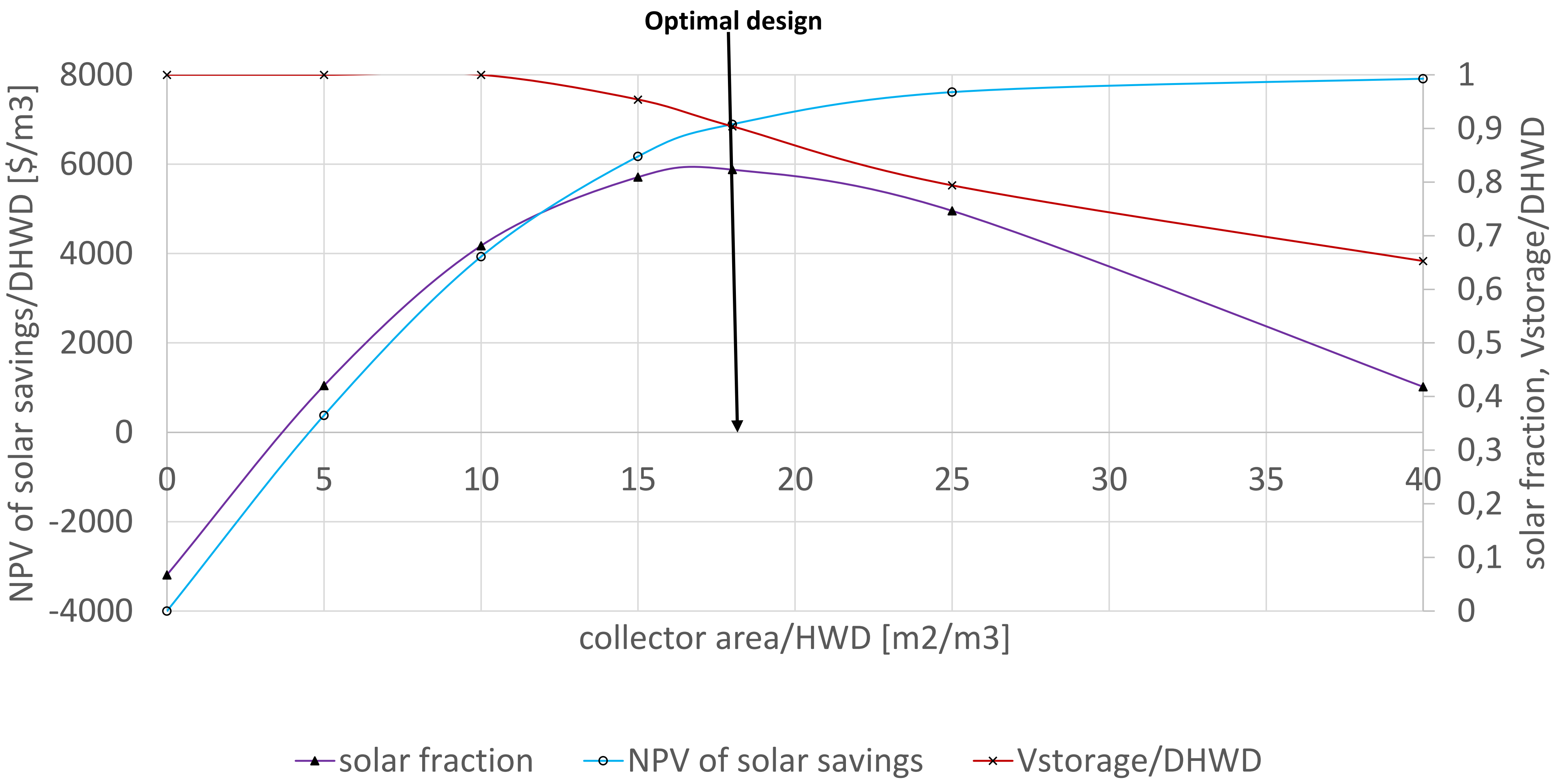
Rank	Collector Type	Cost/area [$/m²]	$F_R\tau\alpha$	F_RU_L [W/m²/°C]	K_0	K_1	K_2	Warranty [years]	Annual Energy [kWh/m²]	Annualized Cost [$/m2]	Energy/$ [kWh/$m²]	Required Collector area [m²/m³]	Solar Fraction
1	**FPC**	**220**	**0.739**	**3.92**	**1.001**	**-0.166**	**-0.125**	**10**	**1128**	**43.15**	**26.1**	**17.0**	**0.91**
2	FPC	303	0.774	3.08	1	0.000	0	10	1399	59.40	23.5	14.0	0.93
3	FPC	242	0.737	4.65	1.002	-0.201	-0.0006	10	1105	47.46	23.3	16.0	0.87
4	FPC	345	0.758	4.14	1.001	-0.287	0.003	10	1198	67.66	17.7	13.5	0.82
5	**ETC**	**175**	**0.409**	**1.68**	**0.999**	**1.383**	**-0.992**	**5**	**895**	**51.90**	**17.2**	**22.0**	**0.93**
6	FPC	347	0.76	6.22	1.001	-0.035	-0.175	10	1068	68.05	15.7	16.0	0.85
7	ETC	433	0.458	1.58	1	1.313	-1.043	15	1065	71.80	14.8	15.5	0.73
8	ETC	361	0.416	1.08	1.011	0.808	-0.33	10	1025	70.80	14.5	13.0	0.84
9	ETC	211	0.406	1.75	1	1.145	-0.606	5	898	62.58	14.4	22.5	0.94
10	ETC	157	0.383	2.04	1.002	-0.043	0.011	5	658	46.57	14.1	26.0	0.86

K_0, K_1 and K_2 are coefficients of the SRCC-data derived angle of incidence modifier function:

$$K_{\tau\alpha} = K_0 + K_1\left(\frac{1}{\cos\theta} - 1\right) + K_2\left(\frac{1}{\cos\theta} - 1\right)^2$$

FPC =Flat Plate Collector; ETC = Evacuated Tube Collector

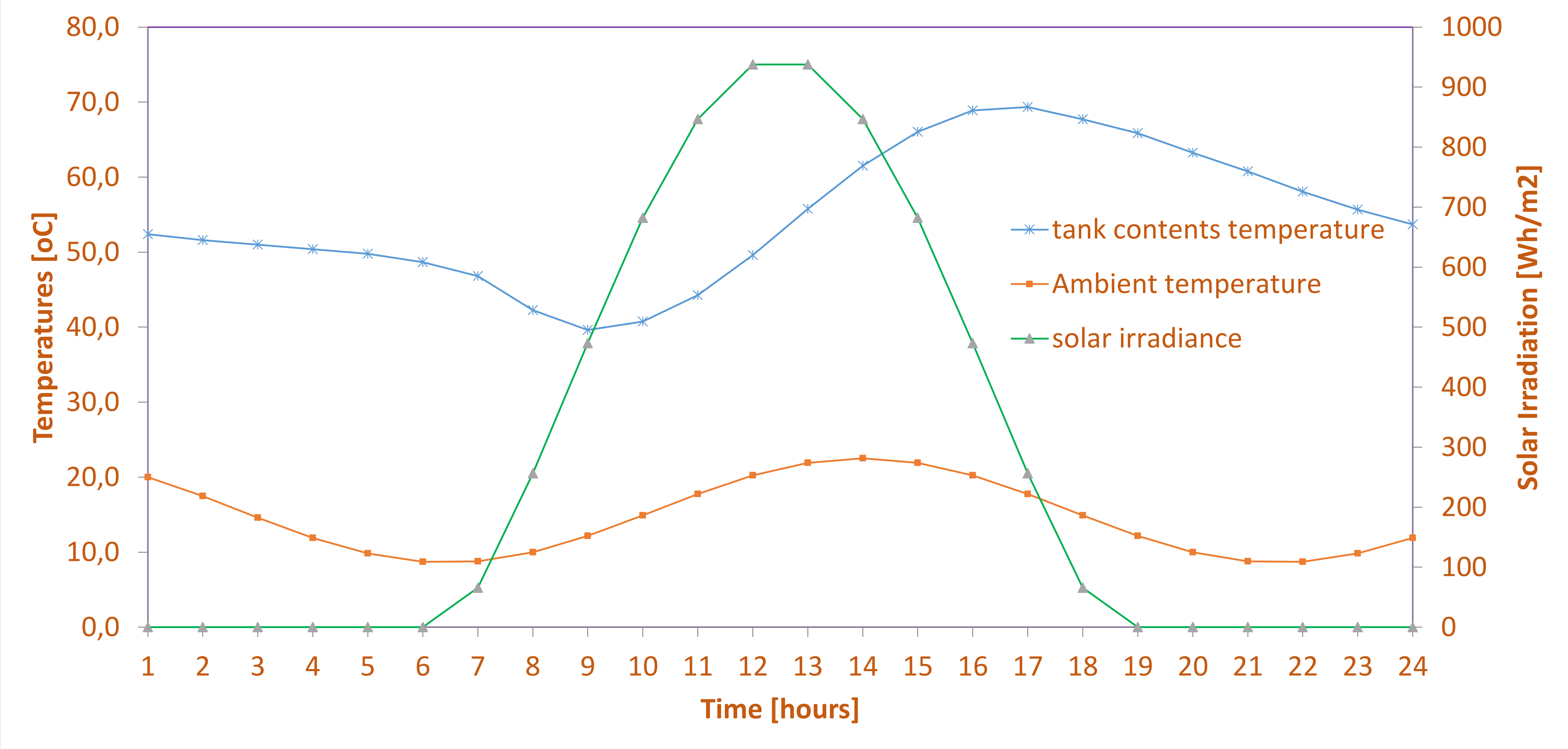
Optimally sized system



Optimal SHWS size/performance parameters: Zimbabwe

- Optimal collector area: 18 m^2/m^3 of hot water demand
- Optimal storage size: 900 litre/m^3 of hot water demand
- Optimal solar fraction: 91 %
- Optimized NPV: $5,881/$m^3$ of hot water demand

Diurnal Performance of Optimal System



Conclusion

- Systematic methodologies are needed in order to make cost-effective decisions about choice and size of solar collector to employ in a solar water heating system.
- The *energy-per-dollar* metric defined in this study, is one such instructive decision-making metric, as it includes all the important-few collector attributes that influence life-cycle cost-effectiveness, i.e. collector *warranty life*; *energy output per unit area* and *cost per unit area*.
- In the study, a sample of SRCC-rated collectors, with differently-attractive attributes such as low cost-per-area, excellent efficiency curves or long warranty lives, were ranked using the energy-per-dollar metric.
- Flat plate type of collectors occupied the top four ranks, for the climatic conditions and load temperature under consideration.
- For the top-ranked collector (26.1 kWh/$), the collector area prescribed in the optimally-sized solar water heating system was 18 m^2/m^3 of hot water demand, the solar fraction 0.91, the storage-demand ratio 0.90 and the NPV was 5,881/m3 of hot water demand.
- The selection and sizing approach used in this study can be generally applied for any temperature operating conditions and for any described climatic conditions.

THANK YOU FOR YOUR ATTENTION!!

“THE SUN DOES NOT BRING A BILL”